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Making Sense of (Multi-)Relational Data

Part IV: Exploration by Descriptive Modelling – Fully Relational Local Approaches

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Fully relational local algorithms

- N-set mining
- RMiner \& variants
- Constraint programming for closed relational sets
- Uncovering the plot

N-set mining

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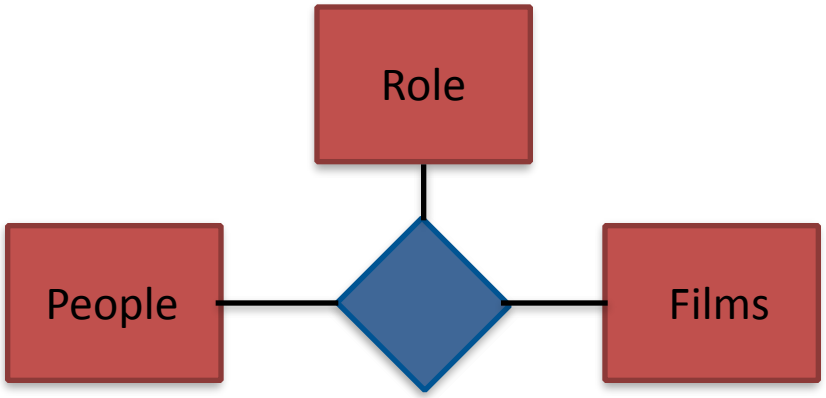
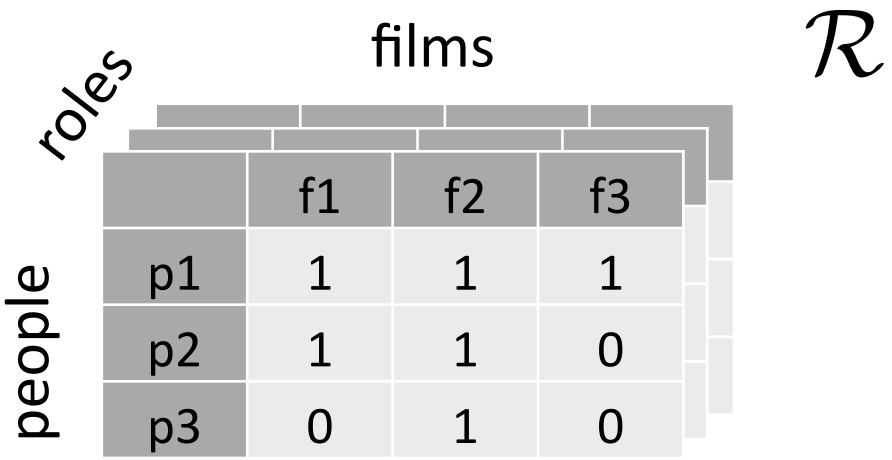
N-set mining (Cerf et al., Trans. Knowl. Discov. Data, 2009



Pattern Syntax, N-sets



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N-sets

$\mathcal{R}$

films

people

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	f2	f3
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

Sets of entities that are complete with respect to the relationship and maximal

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N-sets

$\mathcal{R}$

films

people

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	**f2**	**f3**
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

N-set: {p1, f2, f3, actor, director, producer}

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N-sets

$\mathcal{R}$

films

people

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	**f2**	**f3**
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

N-set: {p1, f2, f3, actor, director, producer}

{p1, f2, actor}$\in \mathcal{R}$ {p1, f2, director}$\in \mathcal{R}$

{p1, f3, actor}$\in \mathcal{R}$ {p1, f3, director}$\in \mathcal{R}$

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Interestingness of N-sets

- No interestingness measure defined.
- Constraints on the number of entities per entity type help to focus on a smaller pattern set.

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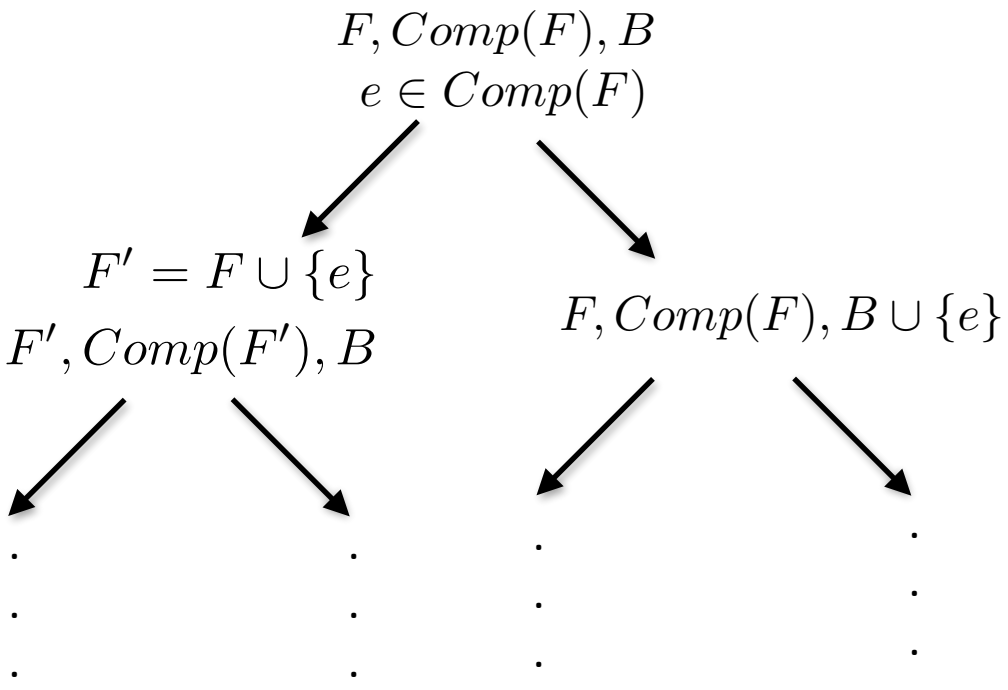
N-set Mining Algorithm - DataPeeler

- Exhaustive Search
- Divide and conquer enumeration strategy
- Use completeness to reduce the search space

DataPeeler

F : current solution

$Comp(F) : \forall e \in E, F \cup \{e\}$ is complete



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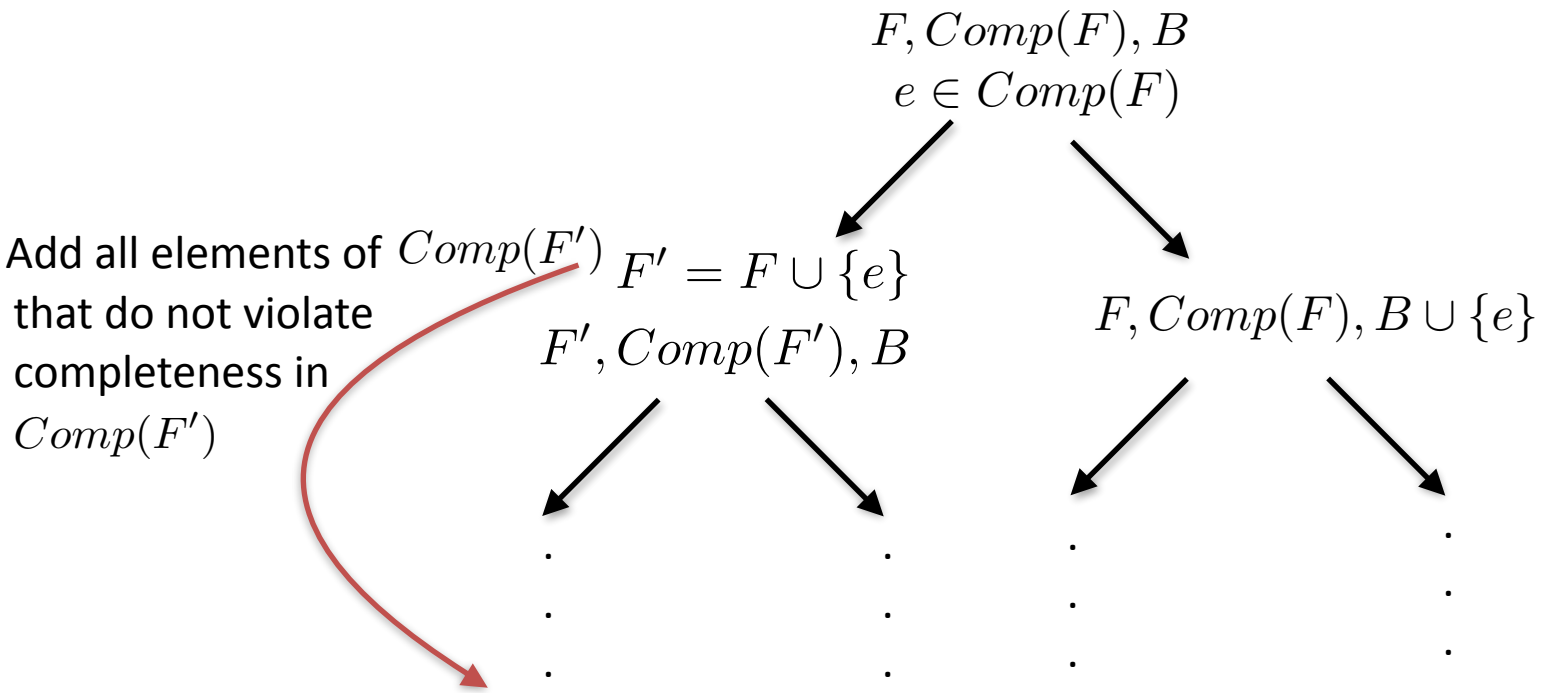
DataPeeler

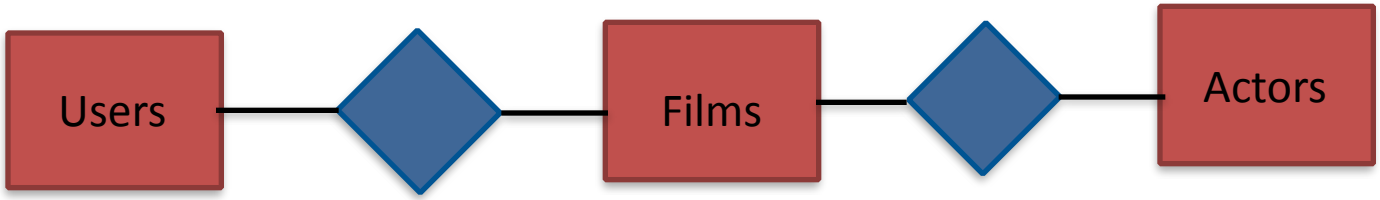
F : current solution

$Comp(F)$: $\forall e \in E, F \cup \{e\}$ is complete



RMiner & variants

RMiner - E.S. et al.,DMKD, 2014



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RMiner - Pattern Syntax (MCCSs)

films

users

	f1	f2	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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MCCSs (Maximal Complete Connected Subsets)

films

users

	f1	f2	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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MCCSs (Maximal Complete Connected Subsets)

Completeness

$$\mathcal{R} = R_{users,films} \cup R_{films,actors}$$

films

users

	f1	f2	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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MCCSs (Maximal Complete Connected Subsets)

Complete Subset: {u1, f1, f2, a1}

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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MCCSs (Maximal Complete Connected Subsets)

Complete Subset: {u1, f1, f2, a1}

$\{u1,f1\} \in \mathcal{R}$
$\{u1,f2\} \in \mathcal{R}$
$\{f1,a1\} \in \mathcal{R}$
$\{f2,a1\} \in \mathcal{R}$

users \ films

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

films \ actors

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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RMiner - Pattern Syntax (MCCSs)

Connectedness

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

RMiner - Pattern Syntax (MCCSs)

Complete Connected Subset (CCS): {u1, f1, f2, a1}

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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RMiner - Pattern Syntax (MCCSs)

Complete **not connected** Subset: {u1, a2}

films

users

	f1	f2	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	**a2**	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

RMiner - Pattern Syntax (MCCSs)

Complete Connected Subset (CCS): {u1, f1, f2, a1}

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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RMiner - Pattern Syntax (MCCSs)

Maximality

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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RMiner - Pattern Syntax (MCCSs)

Maximal Complete Connected Subset (MCCS): {u1, u2, f1, f2, a1, a3}

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	**a3**
f1	1	0	1
f2	1	0	1
f3	0	1	0

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films

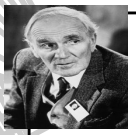
users

43603
30977
46603
24231
58087
48395
48357
33141
45478
38373

genres

Action
Thriller
Adventure

actors



Desmond Llewelyn

Lois Maxwell

RMiner - Interestingness

$$\text{Interestingness}(\gamma) = \frac{\text{Self Information}(\gamma)}{\text{Description Length}(\gamma)}.$$

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Interestingness - Background model

- Maximum Entropy distribution on the data
- Constraints on the expected number of “1s” in every row and every column to be equal to the actual number
- The probability between entities that exist in many relationship instances is going to be high

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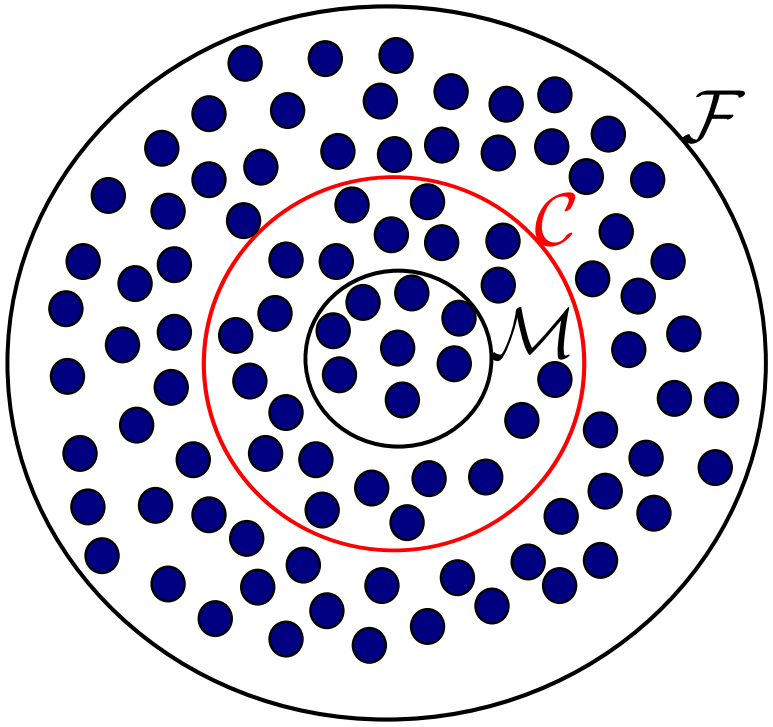
RMiner - Algorithm



$\mathcal{F}$: the set of CCSs

$\mathcal{M}$: the set of MCCSs

$\mathcal{C}$: the set of closed CCSs

$$\mathcal{M} \subseteq \mathcal{C} \subseteq \mathcal{F}$$

Closed CCSs

- Fixpoints of a closure operator

Mapping $\rho : \mathcal{F} \to \mathcal{F}$

Extensivity: $F \subseteq \rho(F)$ for all $F \in \mathcal{F}$

Monotonicity: $\rho(F) \subseteq \rho(F')$ for all $F, F' \in \mathcal{F}$ with $F \subseteq F'$

Idempotence: $\rho(\rho(F)) = \rho(F)$ for all $F \in \mathcal{F}$

RMiner - Algorithm

- Exhaustive search
- Based on divide and conquer algorithmic framework of Boley et al.

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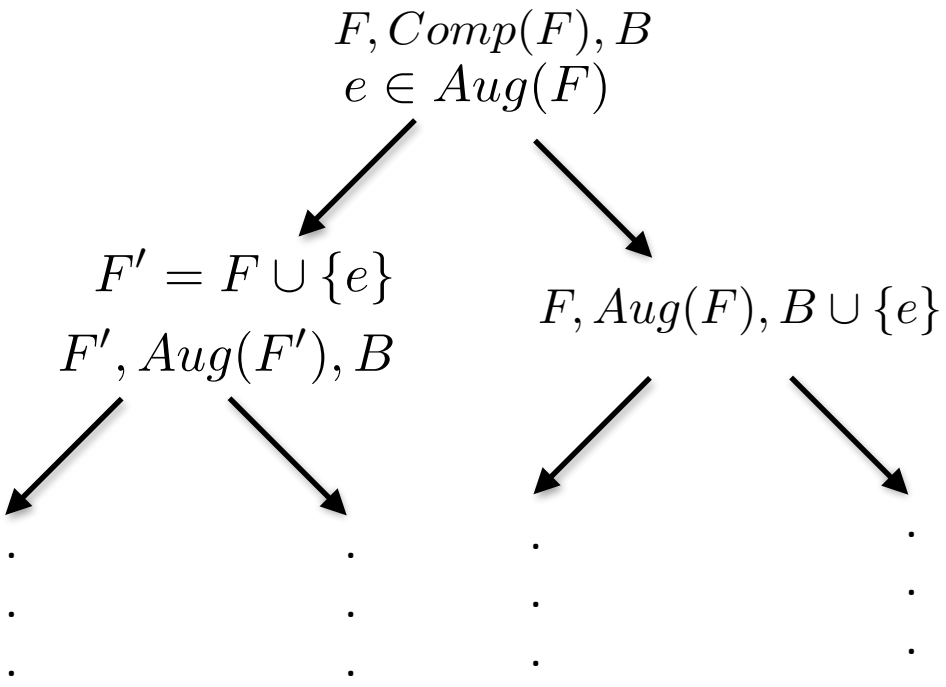
RMiner - Algorithm

F : current solution

$Comp(F) : \forall e \in E, F \cup \{e\}$ is complete

$Aug(F) : \forall e \in E, F \cup \{e\}$ is complete and connected



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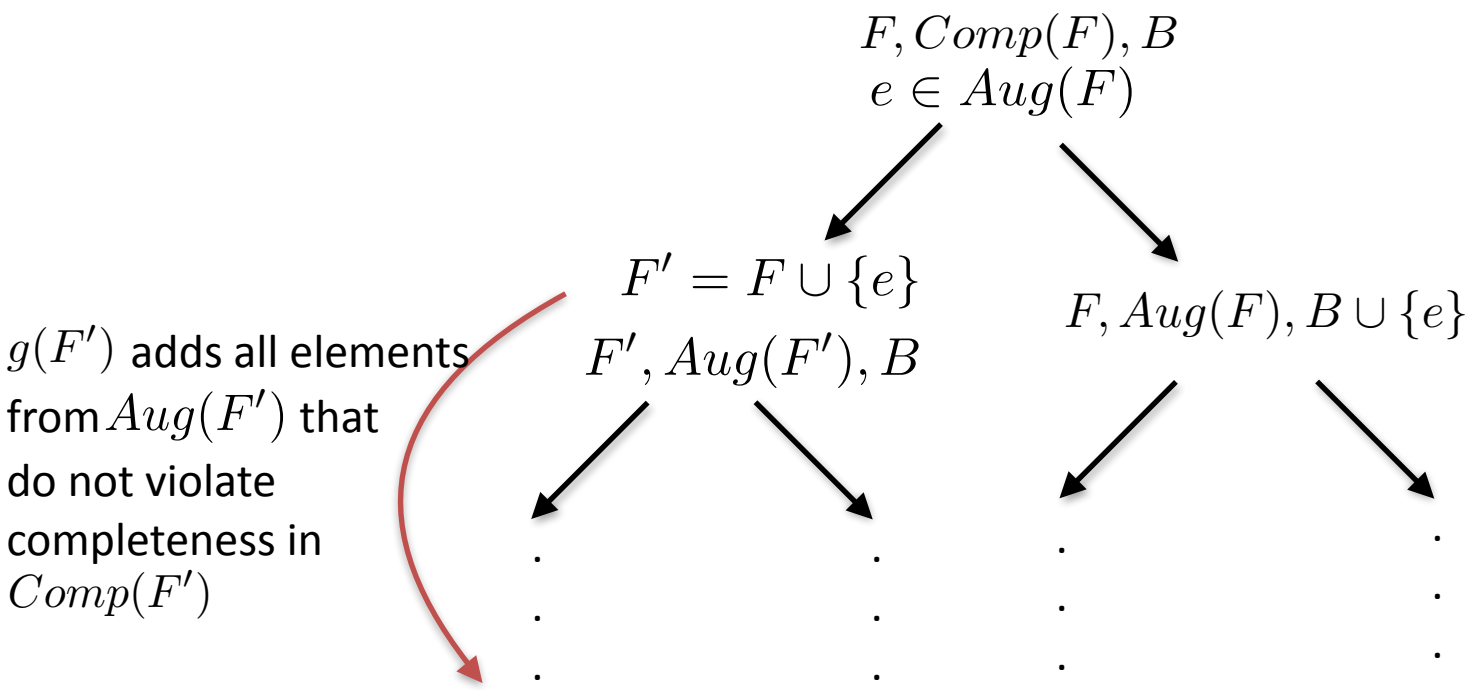
RMiner - Algorithm

F : current solution

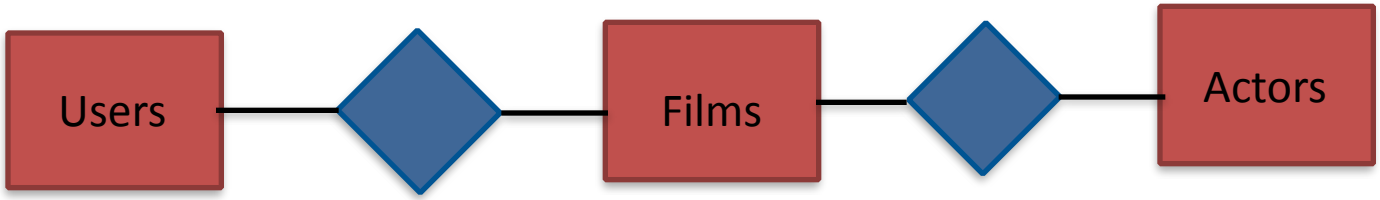
$Comp(F) : \forall e \in E, F \cup \{e\}$ is complete

$Aug(F) : \forall e \in E, F \cup \{e\}$ is complete and connected



A-RMiner

A-RMiner (E.S. et al., DSAA, 2014)



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A-RMiner Pattern Syntax (a-CCSs)

- Unions of MCCSs that are maximal extensions of a closed CCS.

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A-RMiner Pattern Syntax (a-CCSs)

films

users

	f1	f2	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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A-RMiner Pattern Syntax (a-CCSs)

M1 = {u1, u2, f1, f2, a1, a3}

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	**a3**
f1	1	0	1
f2	1	0	1
f3	0	1	0

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A-RMiner Pattern Syntax (a-CCSs)

M2 = {u3, u4, f2, f3}

films

users

	f1	**f2**	**f3**
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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A-RMiner Pattern Syntax (a-CCSs)

a-CCS: M1 U M2 = {u1, u2, u3, u4, f1, f2, f3, a1, a3}

films

users

	f1	**f2**	**f3**
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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A-RMiner - Algorithm

- Same algorithmic framework as RMiner
- Finds approximate patterns at no extra computational cost

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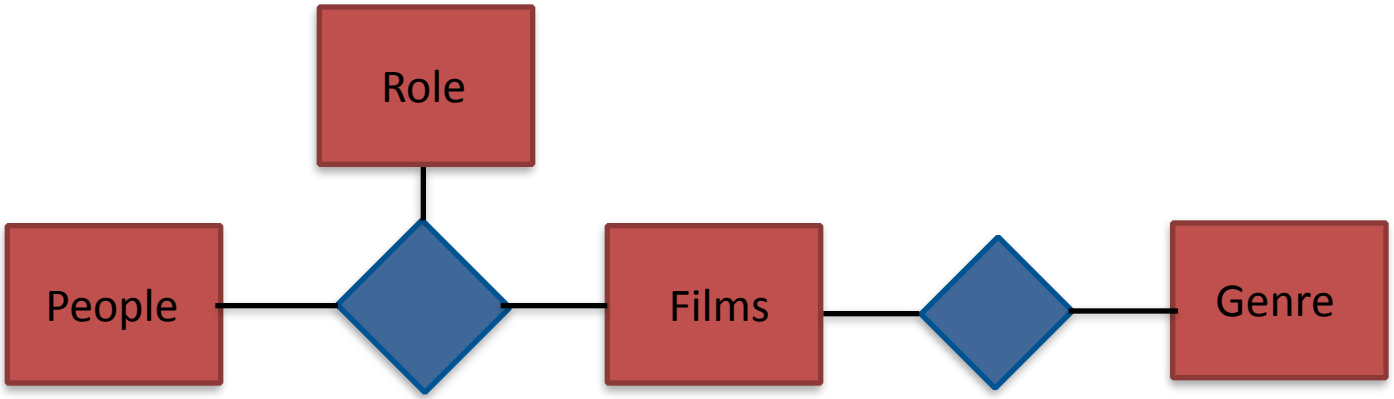
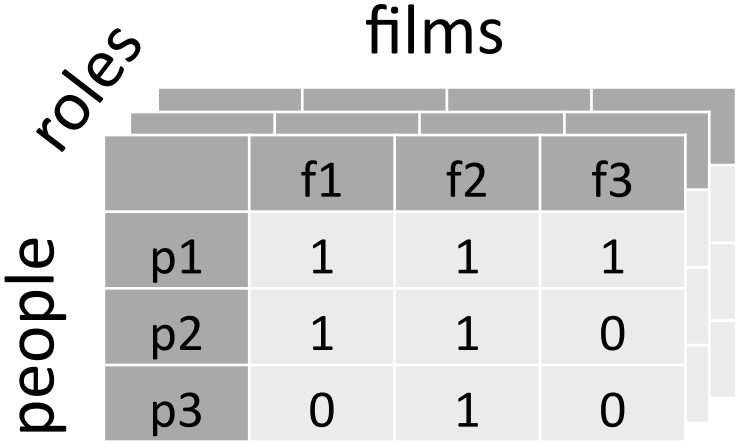
N-RMiner (E.S. et al., Discovery Science 2013)



N-RMiner Pattern Syntax (NMCCSs)



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NMCCSs

films

people

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	f2	f3
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

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NMCCSs

films

people

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	f2	f3
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	f2	f3
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

Completeness: Similar to N-sets with but allowing a subset of the entity types to be in the pattern.

NMCCSs

films

people

	f1	**f2**	f3
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	**f2**	f3
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	f2	f3
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

N-MCCS: {f1, f2, p1, p2, actor, director}

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NMCCSs

films

people

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	1	0	0

actor

	f1	**f2**	**f3**
p1	1	1	1
p2	1	1	0
p3	0	1	0

director

	f1	**f2**	**f3**
p1	0	1	1
p2	0	0	0
p3	0	0	1

producer

N-MCCS: {p1, f2, f3, actor, director, producer}

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N-RMiner - Algorithm

- Same enumeration algorithm
- Different in the way completeness is checked

N-RMiner - Interestingness

- Similar to RMiner

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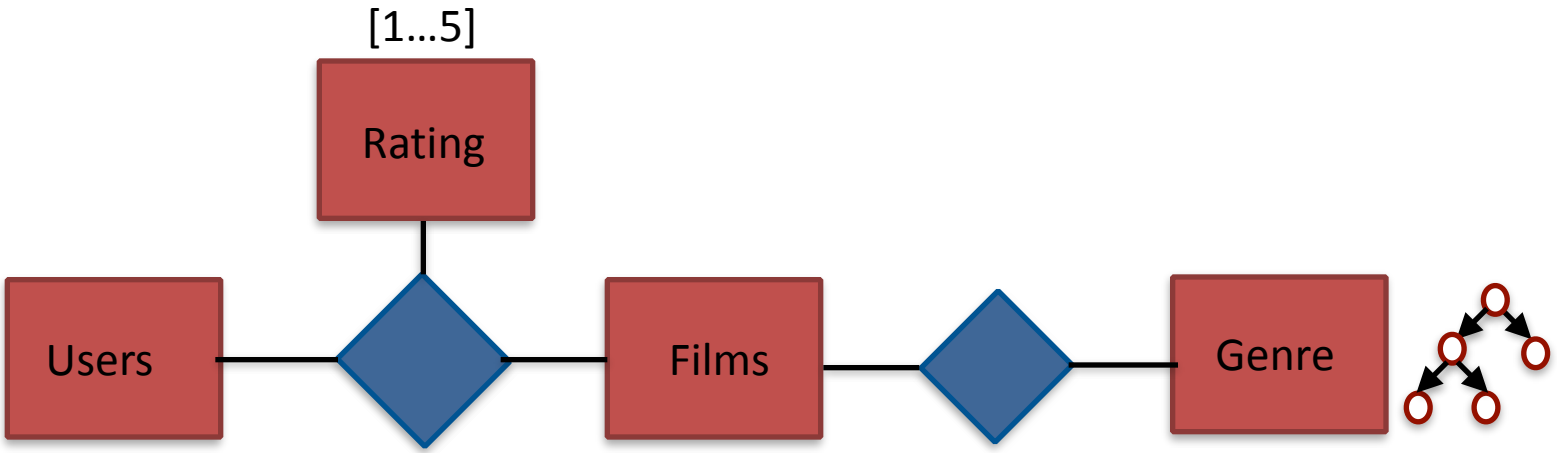
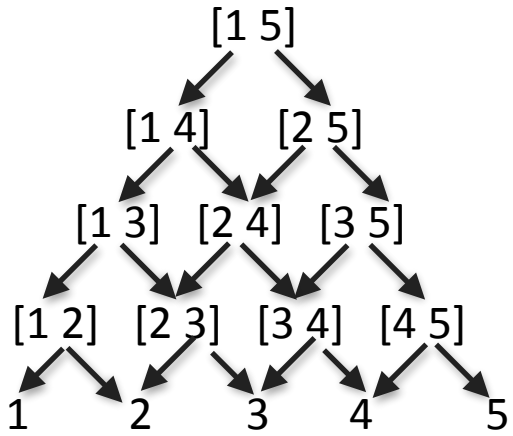
P-N-RMiner (Lijffijt et al., DSAA, 2015)



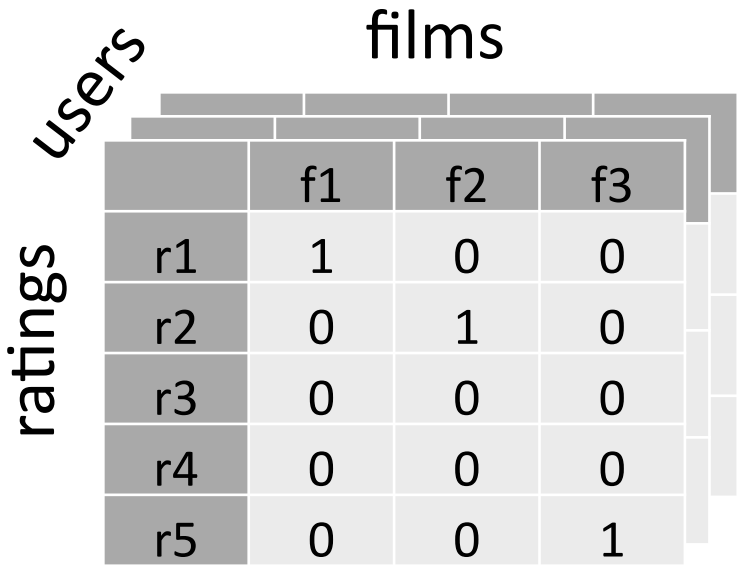
P-N-RMiner, Pattern Syntax

- Represents numerical attributes as partial orders



- It can handle both numerical and hierarchical attributes in the same way

P-N-RMiner, Pattern Syntax (MCCPSs)



	f1	f2	f3
r1	1	0	0
r2	0	1	0
r3	0	0	0
r4	0	0	0
r5	0	0	1

genres

films

	comedy	history	drama
f1	0	1	1
f2	0	1	1
f3	1	0	0

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P-N-RMiner, Pattern Syntax (MCCPSs)

users

films

ratings

	f1	f2	f3
r1	1	0	0
r2	0	1	0
r3	0	0	0
r4	0	0	0
r5	0	0	1

genres

films

	comedy	history	drama
f1	0	1	1
f2	0	1	1
f3	1	0	0

MCCPS: {user1, user2, [1 2], f1, f2, history, drama}

P-N-RMiner, Algorithm

- Same enumeration framework as RMiner
- Uses the partial order to reduce the search space

Constraint programming for closed relational sets

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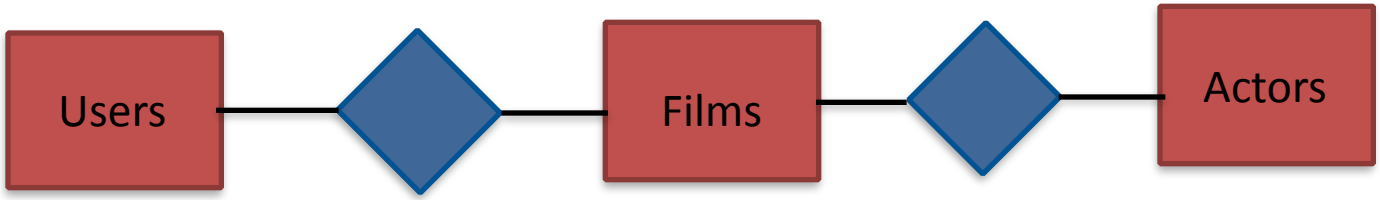
Constraint programming for closed relational sets (Nijssen et al., ICDM Workshops 2011)



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Constraint programming for closed relational sets (Nijssen et al., ICDM Workshops 2011)

- Constraint programming approach
- Defines both pattern syntax and pattern quality constraints
- Pattern syntax is the same as that of MCCSs
- Pattern quality constraints are on the minimum number of entities per entity type

Uncovering the plot

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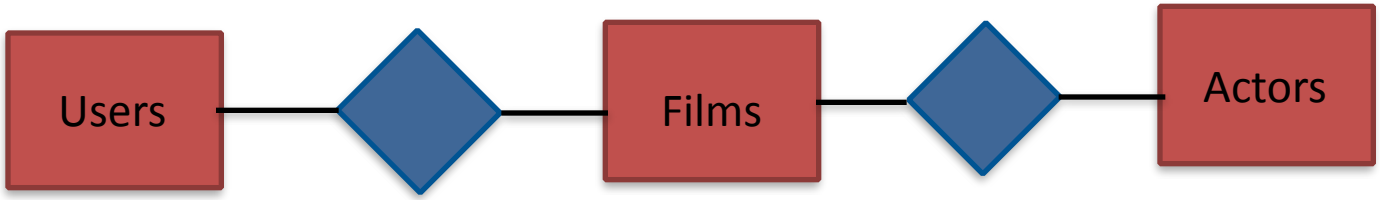
Uncovering the plot (Wu et al. DMKD, 2014)



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Uncovering the plot (Wu et al. DMKD, 2014)

- Approximate multi-relational patterns similar to a-CCSs.
- Defined as chains of bi-clusters such that they overlap on one entity type.
- Works only for binary relationships.

Uncovering the plot - Bi-cluster Chains

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

Uncovering the plot - Bi-cluster Chains

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

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Uncovering the plot - Bi-cluster Chains

films

users

	f1	**f2**	f3
u1	1	1	0
u2	1	1	0
u3	0	1	1
u4	0	1	1

actors

films

	a1	a2	a3
f1	1	0	1
f2	1	0	1
f3	0	1	0

{u1, u2, f1, f2, a1}

Uncovering the plot - Algorithm

- Based on mining all bi-clusters on every relationship first.
- Then greedily combining them to form a chain.

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Uncovering the plot - Pattern interestingness

- Information content with respect to MaxEnt background model
- Different prior knowledge to RMiner and variants
    - Defined over the join of all relationships
    - Has the form of constraints over the area of a tile.

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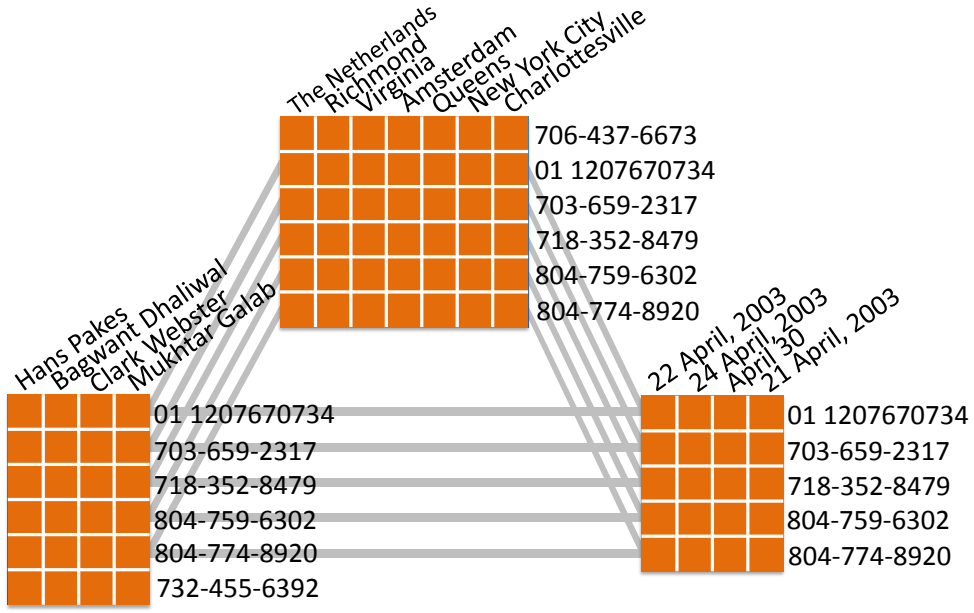
Uncovering the plot - Pattern example



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